Colorado Academic Standards

MATHEMATICS

May 2009
Draft

Colorado Department of Education
Office of Standards and Assessments

cde Improving Academic Achievement

A Call for Feedback

Using the WestEd Colorado Model Content Standards Review as a jumping off point, the mathematics subcommittee spent four months reviewing and revising Colorado's mathematics standards. The process was challenging at times but always invigorating and ultimately rewarding.

The subcommittee relied heavily on national and international benchmarks such the mathematics standards from Singapore, Finland, Massachusetts, and Virginia, as well as national documents such as Achieve's *Mathematics Benchmarks,* College Board's *Standards for College Success*, the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics*, *Benchmarks 2061*, the Guidelines for Assessment and Instruction in Statistical Education for their work.

Integration of 21st century skills and readiness competencies provided a unique challenge to the subcommittee. The intent of the subcommittee is to provide a roadmap for Colorado mathematics teaching and learning that weaves 21st century and readiness competencies into the content of mathematics. This was accomplished by calling out inquiry questions, applications of mathematics to society, and the nature of mathematics.

It is the desire of the standards committee to create standards that not only ensure that graduates from the Colorado P -12 system are ready to apply knowledge, processes, and thinking skills that come from studying mathematics, but also to create standards that both present and future teachers will be excited about teaching and students will enjoy learning. Please help us reach this goal by giving us feedback on the scope, sequence, viability, and overall quality of these standards.

1. Do Colorado's draft standards address the most critical mathematics knowledge and skills and represent the *coherence* of the discipline?
2. Do the draft standards show a solid *progression* of content and skills from grade-to-grade, and level-to-level, and is the content grade appropriate?
3. Do the draft standards represent a level of *rigor* characteristic of our external referents (Singapore, Finland, Massachusetts, and Virginia)?
4. How well does this draft adhere to the principle of "necessary for all, sufficient for some"?
5. Are the draft standards appropriately *focused*, demonstrating that choices have been made about what is most important for students to learn? In other words, have the standards avoided being "a mile wide and an inch deep"?
6. Are the draft standards written in *specific*, *clear* and *measurable* language?
7. How well can these standards be translated into curriculum? Units? Lessons?
8. How do the threads from the prepared graduate competencies follow through to the standards? To the grade level expectations? To the 21st century skills and competencies?
9. How do you envision using the 21st century skills and readiness competencies? How might the document make these important skills actionable?

Respectfully submitted by the Mathematics Content Review Subcommittee Co-Chairs
Michael Brom and Lew Romagnano

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Background of the draft 2009 Colorado Academic Standards

Colorado has had state model content standards in 13 disciplines for 14 years. They have been the broad articulation of the ideas, themes, facts, and agendas which have been of value to this state for more than a decade. In 2007, the State Board of Education recommended a comprehensive revision of these standards for some elemental changes in the direction of these documents. In 2008, the state's legislature affirmed and further articulated the nature of these changes by passing Senate Bill 08-212.

The timeline for revising the content standards in 13 areas is ambitious. State content standards are to be revised and adopted by December 2009. Unlike the time it took to initially create content standards, this one year project was undertaken with select design features in mind.

Senate Bill 08-212 expanded the vision of public education outcomes by including higher education and early childhood education goals into one seamless standards policy.

The vision calls for students to have the knowledge and skills necessary for both the opportunity for college entrance and the capacity of new skill sets in all courses of life.

The four important design features…

This version of state standards will include four changes; 1) adding 21st century skills, 2) ensuring fewer, clearer and higher standards, 3) adding early childhood, postsecondary and workforce readiness expectations, and 4) mastering concepts and skills….not facts.

Design Feature 1: ***21st Century Skills***

The speed with which information, business, culture and new knowledge now moves is a new challenge for all of us. Unlike the historical two decade-long apprenticeships of the Renaissance or the understanding that a person stays in a job for a lifetime, shifts in communication, art, technical sectors and the marketplace now demand that our students be nimble and anticipate problems, solve critical issues and work with others in effective and ethical ways, and function under faster time pressures. Retaining only one body of facts for life does not work anymore. The eight to five workdays have been replaced by twenty-four hour schedules. New skills are needed to thrive under these conditions.

"21st century skills" is a term that most commonly resonates as a way to describe the skills necessary to respond and lead well in a globally-based culture.

Research (21st Century Partnership, SCANS report, EPIC, College Board, etc.) points to the importance of five essential skills. These five include problem solving/critical thinking, information management, collaboration, self direction and innovation. Coloradoans have been surveyed, interviewed and engaged on these five skills and the

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vast majority agrees on their importance. Colorado's draft description of 21st century skills is now linked to the Colorado Department of Education at http://www.cde.state.co.us/index_home.htm. These skills are prominently embedded in the new standards and change the original version from academic content standards to essential concepts and skill standards.

Design Feature 2: ***Fewer, Clearer and Higher***

Standards are written in broad terms in order to capture the volume of facts they represent. Colorado is designing its new expectations based on the best standards in national and international circles. Central to this work is the notion that internationally competitive standards tend to be fewer, higher and clearer than those typically seen in the U.S. today.

- Fewer: The challenge has been to develop standards that reduce lengthy litany of hopes, facts and agendas into essential concepts which are intelligent building blocks grade by grade.
- Clearer: The aim has been to use jargon-free terms that crisply convey knowledge and skill outcomes. Simple language and terms give students and teachers tangible and meaningful endpoints.
- Higher: The aim has been to chart what all students should know and be able to do in order to be successful. Short–changing students with minimal expectations diminishes their life options. These new Colorado expectations are authentic and include goals which invite both a stretch of effort and accomplishment. They convey the message that all students have the capacity to achieve highly.

Design Feature 3: ***Early Childhood and Higher Education Expectations***

To ensure a seamless extension of standards that provides for each level of learning throughout Colorado's entire education system, these standards begin with postsecondary and workforce competencies. They begin with the end in mind. For example, what should every citizen in Colorado regardless of life experience, career, college or military service necessarily know to be considered a successful high school graduate? These big ideas that describe a prepared graduate are listed at the top of each page in the new Colorado academic standards. The Building Blocks of early learning are now coherently added to each content area and represent what is needed in order for a youngster to progress from early understandings to twelfth grade mastery.

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Design Feature 4: ***Concepts….not facts***

Our body of collective knowledge in any discipline grows so rapidly that the concept that schooling is static and a common information transmission system is now untenable. Standards become unwieldy when they attempt to capture the sum of what information students "should" know.

This design feature transforms Colorado academic standards into fewer, crucial *concepts* and *skill* standards that serve to give a mind the essential background, fluency of the topic in depth and the problem solving levers that are the qualities of knowledgeable people in each content area.

The Revision Process

Colorado's standards are being revised using a few deliberate processes. The assumptions behind this approach are:

- These are Colorado's standards. While we invite outside advice, these expectations represent the place where we live.
- The best national and international standards must be used to improve expectations of our kids. We close the gap when we require the quality of what we accept about the worthiness of our student outcomes.
- College opportunities occur when students are stretched beyond mediocre activities and when they do intellectual exercises that model what academic and solution leaders do.
- Research matters. Reinventing what we think about just within our own local circles insults the collective advances others have made.
- Public feedback and improvement informs the next generation of student standards. Different formats were used to gather public feedback:
    - Regional face to face conversation
    - Webinars
    - Listening logs
    - Surveys
    - Stakeholder meetings
    - Colorado experts as subcommittee members
    - Public notes posted with full transparency
    - Professional and association sector outreach
    - Public hearings

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A brief overview of the year's revision process

The beginning of the entire process started with research and gap analysis of benchmarked states and nations which have the best standards. Analysis about the existing strengths and weaknesses simultaneously occurred with the current Colorado standards. Additionally, a study was commissioned which examined the formats and grade span structure of other states' standards.

A stakeholder advisory group was assembled to help define terms, frame the issues, determine grade by grade articulation and select subcommittee members from a pool of untitled applicants. These subcommittees were formed – one for each subject – in order to undertake the revision of standards.

Applicants were solicited from across the state to apply the subject-specific education (early, k-12, and higher education) and business sector expertise. Seven hundred and eighty six people applied to fill 255 unpaid roles. Selection was made by Colorado stakeholders in a name-blind process using the merits of both the application and resumes.

Eight districts that demonstrated early success in revising their local content and skill standards were asked to advise the process.

Advisory committees were identified in P-3, Higher Education, Business, and Education Associations in order to check the rigor and relevance.

Content subcommittees were formed in three phases Phase I: Math, Science, Reading and Writing and Music, Phase II: History, Civics, Geography and Economics, Phase III: Visual Arts, Theatre, Dance, Physical Education/Health and World Language). Each take the research, gap analyses, reports on best benchmarked states and nations, the 21st century skills and a draft of the postsecondary and workforce readiness draft description into consideration in order to revise the content standards of Colorado.

Regional tours after each Phase revision will launch a campaign to gather feedback and suggestions for improvement. The Colorado Department of Education content specialists will identify common themes that emerge from the feedback provided by the field. The first feedback window will be in April, the second in August and the third tour and electronic outreach will be in September.

In June 2009 the Colorado Council of Higher Education and Colorado State Board of Education intend to reach agreement on a working description of postsecondary and workforce readiness. This description may prompt the state to modify the standards drafts.

National experts will also on-going advice and will provide needed editing. They also will provide suggestions of technique.

UCLA's Director of the Center for Research, Evaluation, Standards and Student Testing will compile the complete drafts to write the final version in October 2009.

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Official public hearings will take place in November 2009 before the State Board of Education. These hearings will provide the final recommendations.

The State Board of Education will decide which standards are adopted. This will occur by December 2009.

What can I do to help?

1. First read to understand.
    a. These standards are not like the existing ones
    b. They are not a curriculum or an exhaustive detail of each lesson or fact.
    c. They represent the few, crucial concepts and skills students need to have mastered by the end of each grade. By design, no mention is made of when they actually are taught in the classroom.
2. Learn the new structure.
    a. At the top of each discipline are the final prepared graduate competencies all students should be able to do in twelfth grade. This is the goal or the "end in mind" behind the rest of the text.
    b. The standards are the "buckets" of how we organize the big ideas of one subject of study into those which are related to one another. (For example, pattern finding and algebraic thinking are similar and therefore are organized together within mathematics)
    c. Each grade or grade span now has grade levels of expectation (of mastery), not "benchmarks". These represent a distinct concept and skills a student should know.
    d. Now, each expectation also has an evidence outcome for a student to make meaning of the knowledge and *prove* how they know it. This is intended to engage the student and help them find relevance in the study. The 21st century skills are a part of this evidence element.
3. Examine this document as a prototype. It is a first draft of revised state standards. It is by no means perfect or finished. Does this conceptually work? How would you improve it?
4. Let us know your comments and stay engaged early this year.
    a. Write us your specific ideas about what needs to be kept and what needs improvement at our web page.
    b. Attend a face to face evening town meeting at one of 24 city sites across the state of Colorado.

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One final comment:

These drafts represent a new kind of state standards. It is not a resource or a substitution for curriculum. It is intentionally only the outline of the most crucial concepts and skills students must master at grade level or high school level to be successful for opportunities at higher learning institutions.

It begs for new kinds of professional development, teacher education, state curriculum supports and new assessments.

…And *that* was the genesis of the change mentioned at the beginning of the 2008 Colorado education reform.

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Principles of the Standards Review Process

The Colorado Model Content Standards revision process has been informed by these guiding principles:

- Begin with the end in mind; define what prepared graduates need to be successful using 21st century skills in our global economy.
- Align K-12 standards with early childhood expectations and higher education.
- Change is necessary.
- Standards will be deliberately designed for clarity, rigor, and coherence.
- Standards will be fewer, higher, and clearer.
- Standards will be actionable.

Notable Changes to the Colorado Mathematics Model Content Standards

The most evident changes to the Colorado Math standards result from a change from grade band standards (K-4, 5-8, and 9-12) to grade level expectations. These are explained here in addition to other changes that will be apparent upon comparison between the current math standards and the proposed changes.

1. **Impact of standards articulation by grade level**. The original Colorado Model Content Standards for Mathematics were designed to provide districts with benchmarks of learning for grades 4, 8, and 12. The math standards revision subcommittee was charged with providing more a specific learning trajectory of concepts and skills across grade levels, from early school readiness to post-secondary preparedness. Articulating standards by grade level in mathematics affords greater specificity (clearer standards) in describing the learning path of important mathematics across levels (higher standards), while focusing on a few key ideas at each grade level (fewer standards).

2. **Articulation of high school standards**. High school standards are not articulated by grade level but by standard. This is intended to support district decisions on how best to design curriculum and courses, whether through an integrated approach, a traditional course sequence, or through alternative approaches such as through Career and Technical Education. The high school math standards delineate what all high school students should know and be able to do in order to be well prepared for any post-secondary option. The individual standards are not meant to represent a course or a particular timeframe. All students should be able to reach these rigorous standards within four years. Students with advanced capability may accomplish these expectations in a shorter timeframe leaving open options for study of other advanced mathematics.

3. **Integration of P-2 Council's recommendations**. The math subcommittee has integrated the P-2 Building Blocks document into the P-12 math standards, aligning expectations to a great degree. Important math concepts and skill are clearly defined across these foundational years, detailing expectations to a much greater extent for teachers and parents.

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4. **Standards are written for mastery**. The proposed revisions to standards define mastery of concepts and skills. Mastery means that a student has facility with a skill or concept in multiple contexts. This is not an indication that instruction on a grade level expectation begins and only occurs at that grade level. Maintenance of previously mastered concepts and skills and scaffolding future learning are the domain of curriculum and instruction, not standards.

5. **The processes and procedures of school Algebra have been made more explicit**. A comparison of an original Colorado Math Standard with the proposed revision illustrates this increased level of specificity.
   - Original Standard 2, benchmark 4
     - *Analyze and explain the behaviors, transformations, and general properties of types of equations and functions (for example, linear, quadratic, exponential).*
   - Revised Standard 2, expectations 2 and 3
     - *Analysis of elementary functions and their inverses, by investigating rates of change, intercepts, asymptotes, domain, range, and local and global behavior using all available tools, including technology.*
     - *Application of transformations, including arithmetic combinations (addition, subtraction, and multiplication) and translations (vertical, horizontal, and dilations) to representations of elementary functions using tables, graphs, symbols, text, and geometric models, using all available tools, including technology.*

6. **Explicit evolution of algebra concepts**. The proposed revisions include a more explicit delineation of algebra concepts across grade levels. Algebra concepts develop from elementary through middle school with a rigorous treatment of algebra content in grades 8 and high school.

7. **Design of emphasis on standards across grade levels**. The subcommittee deliberately designed the standards to emphasize specific concepts and skills at different grade levels. This allows teachers to focus on fewer concepts at greater depth than in the past.

8. **Intentional integration of technology use, most notably at the high school level**. Using appropriate technology to allow students access to concepts and skills in mathematics in ways that mirror the 21st century workplace.

9. **Greater focus on Data Analysis, Statistics and Probability across all grade levels**. Information literacy in mathematics involves the ability to manage and make sense of data in more sophisticated ways that in the past. The subcommittee notes this by emphasizing data analysis, statistics, and probability to a greater degree than in the original math standards.

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Below is a quick guide to other changes in the mathematics standards:

Area	Summary of changes	
	Current Standards	**Proposed Revisions**
Number of standards	Colorado has 6 standards in mathematics.	The proposal is to combine standards 1 and 6 and standards 4 and 5.
Names of standards	**Standard 1** Number Sense and Number Relationships **Standard 2** Patterns and Algebra **Standard 3** Data and Probability **Standard 4** Geometry **Standard 5** Measurement **Standard 6** Computation	**Standard 1** Number Sense, Properties and Operations **Standard 2** Patterns, Functions and Algebraic Structures **Standard 3** Data Analysis, Statistics and Probability **Standard 4** Shape, Dimension and Geometric Relationships
Integration of 21st century and post-secondary workforce readiness skills	Not deliberately attended to in original document.	• A design feature of the revision process. • Intentionally integrated into evidence outcomes.
P-2	• Standards articulated for grade band beginning with Kindergarten. • Benchmarks articulated by grade band of K-4 with most geared to upper grades.	• Pre-K included. • Grade level expectations articulated for each elementary grade. • Clear expectations articulated for grades PK-2.
Number of grade level expectations (GLE)	Average of 27 benchmarks per grade level.	Average of 10 grade level expectations per grade level (K-8) with 22 for high school.

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Mathematics Subcommittee Members

Co-Chairs:

Mr. Michael Brom
Middle
Title I math teacher
Douglas County Schools

Mr. Lew Romagnano
Higher Education
Professor of Mathematical Sciences
The Metropolitan State College of Denver

Subcommittee Members:

Ms. Kristine Bradley
Higher Education
Assistant Professor of Mathematics
Pikes Peak Community College

Ms. Kristina Hunt
High School
Mathematics Instructor
Vista Ridge High School
Falcon School District 49

Mr. Greg George
District
K-12 Mathematics Coordinator
St. Vrain Valley School District

Ms. Deborah James
Elementary
Principal at Burlington Elementary
Burlington School District

Ms. Camis Haskell
Elementary
Fifth Grade Classroom Teacher
Monroe Elementary
Thompson School District

Ms. Catherine Martin
District
Director of Mathematics and Science
Denver Public Schools

Mr. Lanny Hass
High School
Assistant Principal
Thompson Valley High School
Thompson School District

Mr. Richard Martinez Jr.
Business
President and CEO
Young Americans Center for Financial Education and Young Americans Bank

Ms. Clare Heidema
Elementary
Senior Research Associate
RMC Research

Ms. Leslie Nichols
Middle
Math Teacher
Lake City Community School

Mr. James Hogan
Elementary
Elementary Math Instructional Coordinator
Aurora Public Schools

Ms. Alicia Taber O'Brien
High School
Mathematics Department Chair
Pagosa Springs High School
Archuleta School District 50

DRAFT

Ms. Kathy O'Sadnick
Middle
Secondary Math Instructional Specialist
Jefferson County Schools

Ms. Kim Pippenger
Elementary
6th Grade Teacher
Pennington Elementary
Jefferson County Schools

Mr. Robert Powers
Higher Education
Associate Professor of Mathematical Sciences
University of Northern Colorado

Ms. Rebecca Sauer
Middle
Secondary Mathematics Coordinator
Denver Public Schools

Mr. James Schatzman
Business
Senior Scientist - Northrop Grumman
Substitute teacher
Aurora and Cherry Creek Public Schools

Ms. Julie Shaw
Elementary
Elementary Math Coordinator
Colorado Springs School District 11

Mr. Jeff Sherrard
Business
Director, Information Technology
Ball Corporation

Ms. T. Vail Shoultz McCole
Pre-K
Instructor
Colorado Community Colleges Online

Ms. Julie Steffen
Pre-K
Early Childhood Special Education Teacher
Invest in Kids
Littleton

Ms. Julie Stremel
High School
Mathematics Teacher and Dept Chair
Aurora Central High School
Aurora Public Schools

Ms. Diane Wilborn
Middle
Assistant Principal
Eagleview Middle School
Academy School District 20

Ms. Julie Williams
High
Assistant Principal
Doherty High School
Colorado Springs School District 11

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Mathematics National Expert Reviewer

Dr. Ann Shannon is a mathematics educator with many decades of experience specializing in standards, assessment, and curriculum. Currently, Ann works as an independent consultant helping states, districts, and schools to better serve the needs of diverse learners of mathematics.

Dr. Shannon was employed as a Research Fellow at the Shell Centre for Mathematics Education, University Nottingham, England before moving her work to the University of California, Berkeley in 1994.

At the University of California, she developed performance assessments for the NSF-funded Balanced Assessment project and the New Standards project. Her 1999 monograph, *Keeping Score*, was published by the National Research Council and drew on her work for Balanced Assessment and New Standards:

In the recent past, Ann has helped Maine, Georgia, and Rhode Island develop academic standards for learning mathematics.

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References used by the mathematics subcommittee

The mathematics subcommittee used a variety of resources representing a broad range of perspectives to inform their work. Those references include:

- *Singapore National Curriculum*
- *Massachusetts Curriculum Framework*
- *Virginia Standards of Learning*
- *Finland – National Core Curriculum*
- WestEd Colorado Model Content Standards Review
- Achieve *Benchmarks for Elementary, Middle, and High School Mathematics*
- Benchmarks 2061
- College Board *Standards for College Success*
- *Guidelines for Assessment and Instruction in Statistics Education (GAISE)*
- *NCTM* Principles and Standards for School Mathematics *and Focal Points*
- *Standards for Success "Understanding University Success"*
- Minnesota Academic Standards, Mathematics K-12

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Colorado Academic Standards
Mathematics

Pure mathematics is, in its way, the poetry of logical ideas. (A. Einstein)

If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding. (Dr. Michael Brown, Nobel Prize Laureate)

In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We all have a vast and rapidly growing trove of information at our fingertips at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

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Prepared Graduate Competencies in Math

The Prepared Graduate Competencies are the Preschool through Grade 12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduates:

- Understand the structure of our number system and all of its subsets. At their most basic level numbers are abstract symbols that represent real world quantities. The real number system is well ordered, dense, and structured. Numbers sets can be infinite, countable and uncountable.

- Understand quantity through estimation, precision, order of magnitude, comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error.

- Are fluent with basic numerical and symbolic facts and algorithms, are able to select and use appropriate (mental math, paper and pencil and technology) methods based on their efficiency, precision, and transparency. Algorithms can be generalized and are based on the properties of real numbers.

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.

- Recognize and make sense of the many ways in which variability, chance, and randomness appear in a variety of contexts.

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.

- Make sound predictions and generations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

- Apply transformation to numbers, shapes, functional representations, and data.

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

- Communicate effective logical arguments, using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

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Readiness Competencies

Students should find enjoyment and beauty in the study of mathematics. Being involved in the discipline of mathematics involves engaging in both the content and processes of mathematics. Mathematical content involves key concepts of number, algebra, data and probability, and geometry. The processes of mathematics include problem solving, representation, communication, connections, and reasoning and proof. Solving meaningful problems in real world contexts allows students to see the relevance of mathematics to their world. Representing patterns and relationships allows students to make sense of and communicate important ideas in mathematics. Collaboration and communication with others allows students to monitor, assess, and make sense of their own thinking and that of others. Making connections within mathematics and to other disciplines and the world allows students to transfer their learning and become flexible, agile thinkers. Engaging in reasoning and proof allows students access to the heart of mathematical thinking.

Three themes are used to describe these important competencies and are interwoven throughout the mathematics standards: *inquiry*, *application of mathematics to society*, *and the nature of mathematics.* These competencies should not be thought of stand-alone concepts but should be integrated throughout the mathematics curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without content to think about, it is equally impossible for students to understand the mathematics without grappling with and investigating

Inquiry. Inquiry is a multifaceted practice requiring students to think and actively pursue understanding. Inquiry demands that students (a) engage in an active process of observation and questioning, (b) investigate to gather evidence, (c) formulate explanations based on evidence, (d) communicate and justify explanations, and (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Applying the Discipline in Society and Using Technology. The hallmark of learning a discipline is the ability to demonstrate the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, and developing, adapting, and refining solutions for the betterment of civilization. The application of a discipline, including the use of technology, enables students to fully engage in and contribute to a global, interdependent society.

Nature of the Discipline. A discipline is defined by the concepts, skills, and processes that are unique to it. These characteristics are enacted through particular habits of the mind, which define the ways of knowing and thought processes. An understanding of the characteristics innate to the discipline allows students to expand their understanding of its unique contributions to society and the greater good of humanity.

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Math in the 21st Century

Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our fast changing world. Today's students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Math is inherently demonstrated in each of Colorado 21st Century Skills, as follows:

Critical Thinking & Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration

Mathematics is a social discipline involving the give and take of ideas between people. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-direction

Doing mathematics requires a productive disposition and self-direction. This involves monitoring and assessing one's mathematical thinking and persisting in search of patterns, relationships, and sensible solutions.

Invention

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element of the expansion both within as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

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Colorado Academic Standards in Math

The Colorado Academic Standards in Math are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their Preschool through Grade 12 experience.

Number Sense, Properties, and Operations. Number sense provides students with a firm foundation in mathematics. Being a student of mathematics involves building a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

Patterns, Functions, and Algebraic Structures. Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Data Analysis, Statistics, and Probability. Data and probability sense provides a student with tools to make meaning of information and uncertainty. Being a student of mathematics involves asking questions and then gathering or using data to answer these questions. Students learn that there are a variety of data analysis and statistics strategies that enable them to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Shape, Dimension, and Geometric Relationships. Geometric sense allows students to comprehend space and shape. Being a student of mathematics involves analyzing the characteristics and relationships of shapes and structures, engaging in logical reasoning, and using tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real-world and throughout mathematics.

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Grade Level Expectations at a Glance by Grade

High School	
Number Sense, Properties, and Operations	1. Analysis of the properties of the system of real numbers, including a comparison of the rational and real number systems. 2. Use of number theory arguments to justify relationships involving whole numbers. 3. Development of an understanding of counting techniques. 4. Application of computation and estimation.
Patterns, Functions, and Algebraic Structures	1. Representation of continuous relations and elementary functions (linear, quadratic, absolute value, power, and exponential functions and their inverses) and of discrete relations and elementary functions (arithmetic and geometric sequences) using tables, graphs, symbols, text, and geometric models. 2. Analysis of elementary functions and their inverses, by investigating rates of change, intercepts, asymptotes, domain, range, and local and global behavior using all available tools, including technology. 3. Application of transformations, including arithmetic combinations (addition, subtraction, and multiplication) and translations (vertical, horizontal, and dilations) to representations of elementary functions using tables, graphs, symbols, text, and geometric models, using all available tools, including technology. 4. Understand equivalent forms of expressions, equations, inequalities, and relations. 5. Solutions to equations, inequalities and systems of equations using all available tools, including technology. 6. Use of elementary functions (linear, quadratic, power, and exponential and their inverses) and their transformations to identify essential quantitative relationships in a situation and to model real-world situations, using all available tools, including technology.

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High School (continued)	
Data Analysis, Statistics, and Probability	1. Design of censuses, surveys, observational and experimental studies to answer statistical questions and understand the types of inferences can legitimately be drawn from each. 2. Evaluation of the quality of observational studies, surveys, and experimental studies. 3. Selection of appropriate methods to collect, organize and analyze data (numerical and categorical, univariate and bivariate) using tables, graphical displays, and numerical summary statistics. 4. Interpretation of results of a study, including inferences and predictions. 5. Experimentation with random phenomena using probability and simulation with technology.
Shape, Dimension, and Geometric Relationships	1. Determination and utilization of the area of irregular shapes, and surface area and volume of cones and pyramids, cylinders and prisms, and spheres. 2. Relationships among two- and three-dimensional geometric figures, including congruence, similarity and symmetry. 3. Utilization of Cartesian coordinate system in geometric contexts. 4. Representation and application of simple transformations of objects in the plane including translations, reflections, rotations and dilations. 5. Validation of geometric conjectures using deduction, proving theorems and reasonable justifications. 6. Indirect measurement of quantities using techniques of algebra or geometry.
Eighth Grade	
Number Sense, Properties, and Operations	1. Demonstration of an understanding the properties of real numbers in problem solving situations. 2. Construction, use and explanation of effective and efficient computation procedures with rational numbers in problem solving situations.
Patterns, Functions, and Algebraic Structures	1. Descriptions of linear pattern, relation, and function relationships using equations, tables, and graphs in problem solving situations, and conversions among these representations. 2. Use of the properties of algebra, equality, and inequality to create equivalent algebraic expressions and solve linear equations and inequalities using a variety of methods, including technology. 3. Application of properties of linear relations and functions to interpret linear situations using algebraic methods, graphs, and technology.

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Eighth Grade (continued)	
Data Analysis, Statistics, and Probability	1. Analysis and interpretation of two-variable data. 2. Recognition of the ways in which statistics can be misused.
Shape, Dimension, and Geometric Relationships	1. Description, analysis and reasoning about angles, parallel and perpendicular lines, and distance on a coordinate plane. 2. Estimation, creation, and usage of direct and indirect measurements to describe and make comparisons.
Seventh Grade	
Number Sense, Properties, and Operations	1. Construction, use and explanation of equivalent representations (including models) of integers and positive rational numbers. 2. Construction, use, modeling, and explanation of effective and efficient computation procedures with integers and positive rational numbers in problem-solving situations. 3. Apply proportional reasoning to solve for an unknown part in problem solving situations.
Patterns, Functions, and Algebraic Structures	1. Description of linear patterns, relations and functional relationships using equations, tables, and graphs in problem solving situations.
Data Analysis, Statistics, and Probability	1. Analysis and interpretation of one-variable data regarding shape, center, and spread. 2. Analysis of data using concepts of relative frequency.
Shape, Dimension, and Geometric Relationships	1. Estimation and calculation of surface area, volume, circumference of circle, and area of a circle. 2. Application of ratio, proportion, and similarity in problem-solving situations
Sixth Grade	
Number Sense, Properties, and Operations	1. Construction, use and explanation of multiple and equivalent representations (including models) of positive rational numbers. 2. Construction, use, and explanation of effective and efficient estimation strategies and computation procedures with positive rational numbers in problem solving situations. 3. Comparison of quantities using ratios and rates.
Patterns, Functions, and Algebraic Structures	1. Use of a variable as a representation for unknown quantities. 2. Analysis and description of patterns, with and without variables, using words, tables, and graphs.

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Seventh Grade (continued)	
Data Analysis, Statistics, and Probability	1. Formulation of questions and collection of data. 2. Employment of counting strategies to determine sample space. 3. Exploration of the concept of chance using experimental and theoretical probability.
Shape, Dimension, and Geometric Relationships	1. Identification, comparison, conjecturing, and reasoning about properties of two-dimensional shapes. 2. Usage of equivalent measurements for length, capacity, and weight. 3. Solve problems that require attention to both approximation and precision of measurement.
Fifth Grade	
Number Sense, Properties, and Operations	1. Application of number theory concepts and operational properties to solve problems. 2. Equivalence, comparison, ordering, and computation of fractions, decimals, and percents. 3. Fluently use and explanation of efficient procedures to solve multi-digit whole number multiplication and division problems.
Patterns, Functions, and Algebraic Structures	1. Description, display, and analysis of patterns and relationships using a variety of tools including words, tables, graphs and technology. 2. Analysis of relationships to identify how a change in one number results in a change in another number using all four operations.
Data Analysis, Statistics, and Probability	1. Formulation of questions, collection, construction, analysis, and interpretation of data displays to solve problems or communicate results. 2. Generation and analysis of data from chance devices and events.
Shape, Dimension, and Geometric Relationships	1. Identification, description, and analysis of spatial relationships and properties of two and three-dimensional figures. 2. Selection and usage of appropriate tools and units for estimating and measuring to solve problems.

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Fourth Grade	
Number Sense, Properties, and Operations	1. Extension of understanding of base-ten number system to whole numbers to 100,000 and decimals through hundredths. 2. Fluent use and explanation of efficient procedures to solve multi-digit whole number multiplication and division problems. 3. Representation, comparison, and order of common fractions and decimals (for example, 1/2, 1/3, 1/4, 3/4, 0.5, .25) through the use of models and drawings.
Patterns, Functions, and Algebraic Structures	1. Recognition, reproduction, extension, creation, or description of patterns and sequences that are formed using a variety of materials including manipulatives and numbers. 2. Demonstration of understanding of patterns and relationships by solving for missing elements or unknown quantities in problems and simple equations. 3. Observation and explanation about how a change in one quantity can produce a change in another.
Data Analysis, Statistics, and Probability	1. Formulation of questions, collection, construction, analysis, and interpretation of data displays to solve problems or communicate results. 2. Solutions to problems involving combinations 3. Generation, analysis, and representation of possibilities.
Shape, Dimension, and Geometric Relationships	1. Comparison and classification of geometric figures. 2. Solutions to problems using geometric relationships, transformations, and spatial reasoning. 3. Selection and usage of appropriate tools and units for estimating and measuring to solve problems.
Third Grade	
Number Sense, Properties, and Operations	1. Representation, comparison, and order of whole numbers and common fractions. 2. Extension of understanding of base-ten number system to whole numbers to 10,000. 3. Development, discussion, and use of efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers. 4. Demonstration of conceptual understanding of multiplication and division with single digit factors.
Patterns, Functions, and Algebraic Structures	1. Identification, description, and extension of patterns formed using a variety of materials and manipulatives. 2. Recognition, understanding, and use of patterns to solve problems

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Third Grade (continued)	
Data Analysis, Statistics, and Probability	1. Collection, construction, analysis, and interpretation of data displays to solve problems or communicate results from experiments or surveys. 2. Generation of data and predictions using chance devices.
Shape, Dimension, and Geometric Relationships	1. Recognition of basic attributes and properties of geometric figures and usage of these to describe, analyze, and classify shapes. 2. Selection and usage of appropriate tools and units for time, weight, temperature, and precise linear measurements.
Second Grade	
Number Sense, Properties, and Operations	1. Demonstration of place-value concepts to 1,000 through their relationships to counting, ordering, and grouping. 2. Develop, discuss, and use accurate methods to add and subtract two-digit whole numbers. 3. Demonstration of the understanding of commonly-used fraction concepts.
Patterns, Functions, and Algebraic Structures	1. Identification of patterns and relationships and use of that information to solve a problem using a variety of materials. 2. Recognition, description, reproduction, extension, and creation of increasingly complex patterns using a variety of materials including numbers.
Data Analysis, Statistics, and Probability	1. Collection, generation, reading, analysis, and interpretation of displays of data. 2. Generation of data and predictions using chance devices
Shape, Dimension, and Geometric Relationships	1. Application of spatial relationships and properties to describe and transform geometric figures. 2. Selection and usage of appropriate tools and units for measurements using non-standard and standard measures.
First Grade	
Number Sense, Properties, and Operations	1. Demonstration of place-value concepts to 100 through their relationships to counting, ordering, and grouping. 2. Demonstration of the meaning of addition and subtraction, and their relationships. 3. Recognition of part/whole relationship with common fractions.
Patterns, Functions, and Algebraic Structures	1. Creation and extension of a patterns based on rules. 2. Recognition of patterns and usage of that information to solve a problem.
Data Analysis, Statistics, and Probability	1. Generation, reading, analysis, and interpretation of displays of data.

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First Grade (continued)	
Shape, Dimension, and Geometric Relationships	1. Usage of part-whole relationships, attributes, and properties to compose and decompose shapes. 2. Recognition and description of shapes. 3. Comparison and ordering of objects and events according to measurable attributes.
Kindergarten	
Number Sense, Properties, and Operations	1. Representation, comparison, and order of whole numbers less 20. 2. Composition and decomposition of whole numbers through 10; join and separate sets with manipulatives or models. 3. Communication of reasonable estimates in real world number activities.
Patterns, Functions, and Algebraic Structures	1. Recognition, description, reproduction, extension, and creation of simple patterns using a variety of materials including numbers. 2. Description of basic relationships with numbers to 10.
Data Analysis, Statistics, and Probability	1. Categorization and classification of objects and information 2. Description and comparison of data displays.
Shape, Dimension, and Geometric Relationships	1. Recognition, naming, description and construction of basic shapes. 2. Comparison and ordering of objects according to measurable attributes.
PreK	
Number Sense, Properties, and Operations	1. Counting and grouping objects and numbers. 2. Use of numbers and counting as a means for solving problems.
Patterns, Functions, and Algebraic Structures	1. Recognition and creation of patterns.
Data Analysis, Statistics, and Probability	1.
Shape, Dimension, and Geometric Relationships	1. Recognition and creation of shapes and awareness of position in space. 2. Measurement of time and quantity.

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Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Being a student of mathematics involves building a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

Prepared Graduate Competencies

The Prepared Graduate Competencies are the Preschool through Grade 12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in Mathematics standards are:

1. Understand the structure of our number system and all of its subsets.
2. Understand quantity through estimation, precision, order of magnitude, and comparison.
3. Fluency with basic symbolic facts and algorithms and selection and use of appropriate methods.
4. Make relative and absolute comparisons between quantities.
5. Recognize and make sense of the ways variability, chance, and randomness appear.
6. Solve problems and make decisions by understanding, explaining, and quantifying variability in data.
7. Understand that equivalence is a foundation of mathematics.
8. Make sound predictions and generations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
9. Apply transformations to numbers, shapes, functional representations, and data.
10. Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.
11. Communicate an effective, logical argument using mathematical justification and proof.
12. Use critical thinking to recognize problematic situations, create a model, and present and defend solutions.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

1. Analysis of the properties of the system of real numbers, including a comparison of the rational and real number systems.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
(This expectation will be assessed under Standard 2) a. Explain that between any two rational numbers there is a countable and infinite number of rational numbers and that between any two irrational numbers there is an uncountable and infinite number of irrational numbers. b. Explain why the order of operations is a universal convention for evaluating expressions. c. Use distributive, associative, commutative, and inverse properties to analyze the characteristics of number systems.	**Inquiry:** • How dense is the number line? • How many rational numbers are there? • Is there only one meaning of infinite?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Student use real world contexts to model the meanings for real numbers. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

2. Use of number theory arguments to justify relationships involving whole numbers.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
These evidence outcomes will be assessed under Standard 2. a. Develop and justify conjectures about relationships involving whole numbers. b. Develop and justify conjectures about the distributive, associative, commutative, and inverse properties of the whole number system.	**Inquiry:** • Is the sum of two odd numbers always even? • Is the product of two odd numbers always odd? • What makes a conjecture testable? • What relationships exist between numbers? • What value is there to finding relationships with whole numbers?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Student use real world contexts to model the meanings for real numbers. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

3. Development of an understanding of counting techniques.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
This evidence outcome will be assessed in Standard 3. a. Use combinatorics (Fundamental Counting Principle, permutations and combinations) to solve problems in real world contexts.	**Inquiry:** • What are efficient methods for counting combinations? • Why do counting methods work?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the meanings for real numbers. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

4. Application of computation and estimation.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Use appropriate computation methods that encompasses estimation, calculation, and degree of precision. b. Perform operations (addition, subtraction, multiplication, and division) on numbers written in scientific notation with technology. c. Explain the difference between the number expression $n/0$, where $n \neq 0$ and 0/0.	**Inquiry:** • Why do mathematicians use scientific notation? • Why can't numbers be divided by zero?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the meanings for real numbers. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

1. Demonstration of an understanding the properties of real numbers in problem solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Compare and order sets of integers and rational numbers that are expressed as fractions, decimals, or percents. b. Given a whole number from 0 - 100, determine whether it is a perfect square or find the two consecutive whole numbers between which its square root lies. c. Approximate the location of square roots on a number line.	**Inquiry:** • How can different quantities be expressed depending on the desired degree of precision? • How are fractions, decimals and percents used to make comparisons? • How does one decide whether to express a quantity as a fraction, decimal or percent?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model computation procedures with rational numbers. • Students use number sense with rational numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

2. Construction, use and explanation of effective and efficient computation procedures with rational numbers in problem solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Add, subtract, multiply and divide rational numbers including integers, positive and negative fractions and decimals. b. Apply computational methods to solve multi-step application problems involving percents and rational numbers.	**Inquiry:** ▪ How do operations with rational numbers compare to operations with whole numbers? ▪ How do you know if a computational strategy is sensible?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the integers and positive rational numbers. • Students use equivalent representations of integers and positive rational numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

1. Construction, use and explanation of equivalent representations (including models) of integers and positive rational numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Read, write, locate on number line, compare and order integers and positive rational numbers. b. Apply the definition of absolute value with integers. c. Express large and small numbers using scientific notation.	**Inquiry:** • Why are there negative numbers? • What are the ways that positive rational numbers can be represented? • How do we compare and contrast numbers?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model integers and positive rational numbers. • Students use equivalent representations of integers and positive rational numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

2. Construction, use, modeling, and explanation of effective and efficient computation procedures with integers and positive rational numbers in problem-solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Simplify numeric expressions using order of operations. b. Add, subtract, multiply, and divide integers. c. Use mental math and estimation strategies to solve problems involving percents. d. Solve problems involving percent of a number, discounts, taxes, percent increase, and percent decrease.	**Inquiry:** • How do operations with rational numbers compare to operations with integers? • How do you know if a computational strategy is sensible?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving numbers. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model computation procedures with integers and positive rational numbers. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

3. Apply proportional reasoning to solve for an unknown part in problem solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use between and within ratio relationships to solve for a missing value in a proportion. b. Model proportional relationships with bar models, ratio tables, and similar figures. c. Explain the difference between a ratio, rate, and unit rate. d. Use unit rates as a method of comparison.	**Inquiry:** • What information can a relative comparison give that an absolute comparison cannot? • How can comparisons be made using ratios? • Why are ratios needed? • How can we use proportions to solve problems? • How do you know when a proportional relationship exists?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model proportional relationships. • Students use proportional relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

1. Construction, use and explanation of multiple and equivalent representations (including models) of positive rational numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Read, write, compare, convert between and order positive rational numbers in a variety of forms including proper and improper fractions, mixed numbers, decimals, and percents. b. Express whole numbers as products of prime factors with exponents and use prime factorization to find the greatest common factor and least common multiple of two numbers. c. Express the quotient and remainder of a whole number division problem (a/b or a ÷ b) using fractions, terminating decimals, or repeating decimals. d. Locate positive fractions and decimals on a number line.	**Inquiry:** • What are the ways that positive rational numbers can be represented? • Why is there no limit to the number of ways a quantity can be represented? • How do we compare and contrast numbers?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model positive rational numbers. • Students use equivalent representations of positive rational numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

2. Construction, use, and explanation of effective and efficient estimation strategies and computation procedures with positive rational numbers in problem solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Add, subtract, multiply and divide positive fractions, decimals, and combinations of fractions and decimals. b. Solve multi-step word problems involving fractions, decimals and whole numbers. c. Estimate sums, differences, products and quotients of rational numbers using common fractions, common decimals, and whole numbers. d. Compare and round positive numbers from thousandths through millions.	**Inquiry:** • How do fraction and decimal operations affect numbers? • How do you know if a computational strategy is sensible?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model computation procedures with positive rational numbers. • Students use number sense using positive rational numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

3. Comparison of quantities using ratios and rates.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Apply multiplicative reasoning to create equivalent fractions and reduce fractions to simplest form. b. Express the comparison of two whole number quantities using differences, part to part ratios, and part to whole ratios in a real context. c. Compute unit rates in real world situations involving mixtures, concentrations, and distance-time relationships.	**Inquiry:** • How can comparisons of quantities help us understand relationships? • Why is there no limit to the number of ways a fraction can be represented?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model ratios and rates. • Students use ratios and rates to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

1. Application of number theory concepts and operational properties to solve problems.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Apply concepts of squares, primes, composites, factors, and multiples to solve problems. b. Use the identity, associative, commutative, and distributive properties to solve problems. c. Describe and use divisibility rules for 2, 3, 4, 5, 6, 9, and 10 to solve problems. d. Represent numbers to 1,000,000 with expanded notation and exponents. e. Select and justify estimation strategies based on a problem's need for accuracy.	**Inquiry:** • Why do we classify numbers? • How can I tell if an estimate is reasonable? • How do number theory concepts help me to navigate the number system and solve problems? • How can I use operational properties to solve properties? • What are simple ways to determine divisibility without dividing?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model number theory concepts and properties. • Students use number theory concepts and properties to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

2. Equivalence, comparison, ordering, and computation of fractions, decimals, and percents.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Find equivalent forms of fractions, decimals, and percents using models, drawings, and computational strategies. b. Compute changes in price with proportional units. c. Model addition, subtraction, and multiplication of fractions, decimals, and percents.	**Inquiry:** • If a quantity can be represented as a fraction, decimal, or percentage, which is the most appropriate and why? • What are the ways to model operations with fractions, decimals, and percents? Do some work better than others?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model equivalence, comparison, and computation. • Students use different representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

3. Fluently use and explanation of efficient procedures to solve multi-digit whole number multiplication and division problems.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use flexible methods of computing including student-generated and standard algorithms to multiply and divide numbers having two-digit factors or divisors b. Model multiplication and division using area, linear, and grouping models. c. Interpret remainders and select the most useful form of the quotient in division problems. d. Select and use appropriate methods to estimate products and quotients or calculate them mentally depending on the context and numbers involved.	**Inquiry:** • How are multiplication and division related? • What makes one strategy or algorithm better than another?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the procedures for whole number multiplication and division. • Students use multiplication and division procedures to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

1. Extension of understanding of base-ten number system to whole numbers to 100,000 and decimals through hundredths.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Read and write numbers from 1 to 100,000. b. Identify and explain the value of any given digit in a five digit number. c. Read and write numbers to the hundredths place. d. Identify the value of any given digit in a number involving decimals to the hundredths place.	**Inquiry:** • How and why does the position of a digit in a number affect its value? • How do decimal fractions follow the rules of place value?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the meanings for whole numbers and decimal numbers. • Students use whole numbers and decimal numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

2. Fluent use and explanation of efficient procedures to solve multi-digit whole number multiplication and division problems.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use flexible and efficient methods of computing including student-generated strategies and standard algorithms to solve three or four digit by one-digit multiplication or division problems. b. Estimate using strategies such as front end or rounding to justify the reasonableness of solutions to problems c. Demonstrate fluency with multiplication facts and their related division facts 0 to 12. d. Explain why multi-digit multiplication and division procedures work based on place value properties and use them to solve problems.	**Inquiry:** • When is an estimate better than an exact answer? • How close is close enough in an estimate? • How can the knowledge of place value help with multiplication and division of large numbers?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the procedures for multi-digit multiplication and division problems. • Students use efficient procedures for multiplication and division to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

3. Representation, comparison, and order of common fractions and decimals (for example, 1/2, 1/3, 1/4, 3/4, 0.5, .25) through the use of models and drawings.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Solve problems using fractions with denominators through 10, including like and unlike denominators. b. Estimate and justify the reasonableness of solutions to problems. c. Demonstrate equivalent fractions and decimals using drawings and models.	**Inquiry:** • How can different fractions represent the same quantity? • What are the ways to model fractions?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model equivalence, comparison, and computation. • Students use equivalent representations of numbers to communicate solutions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

1. Representation, comparison, and order of whole numbers and common fractions.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use drawings, models, and numerals to represent fractions (halves, thirds, fourths, sixths, eighths) based on a whole shape, set, or number line. b. Use number sense to estimate and justify the reasonableness of solutions to problems involving fractions. c. Recognize that equivalent fractions are two ways of describing the same quantity.	**Inquiry:** • How can a whole number be represented in different, equivalent forms? • How can a fraction be represented in different, equivalent forms? • How do we show part of something?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model whole numbers and fractions, and their equivalents. • Students use representations of whole numbers fractions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

2. Extension of understanding of base-ten number system to whole numbers to 10,000.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Read and write numbers to 10,000. b. Identify and explain the value of any given digit in a four-digit number. c. Generalize the change represented when moving from one place to another place in a number.	**Inquiry:** • How do patterns in our place value system help us compare whole numbers? • How does the position of a digit in a number affect its value?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model quantities using place value concepts. • Students use place value concepts to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

3. Development, discussion, and use of efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use number sense to estimate and justify the reasonableness of solutions to problems. b. Use flexible methods of computing, including student-generated strategies and standard algorithms. c. Estimate using strategies such as front end estimation or landmark numbers.	**Inquiry:** • What makes a computational method efficient? • What makes a good estimate? • How do you know when a computational method works?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the concepts of addition and subtraction. • Students use representations of addition and subtraction to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

4. Demonstration of conceptual understanding of multiplication and division with single digit factors.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Demonstrate fluency with multiplication and division facts with single digit factors of 0, 1, 2, 3, 5, and 10. b. Recognize relationships between related facts. c. Explain the relationship between multiplication and division. d. Represent multiplication and division problems with drawings, models, number sentences, and stories.	**Inquiry:** • How are multiplication and division related? • How can you use a multiplication or division fact to find a related fact? • How can multiplication and division be modeled?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model equivalence, comparison, and computation. • Students use equivalent representations of numbers to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

1. Demonstration of place-value concepts to 1,000 through their relationships to counting, ordering, and grouping.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Read and write numbers to 1,000 and identify the place value of three-digit digit numbers. b. Describe relationships between ones, tens, and hundreds. c. Explain the value of a digit in a three-digit number. d. Order a collection of whole numbers.	**Inquiry:** • What do I know about this quantity? How is it related to ones, tens and hundreds? • How do patterns in our place value system help us compare whole numbers? • How does the position of a digit in a number affect its value?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model quantities using place value concepts. • Students use representations of numbers as hundreds, tens and ones to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

2. Develop, discuss, and use accurate methods to add and subtract two-digit whole numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Demonstrate fluency with basic addition and subtraction facts to sums of twenty. b. Find value of a collection of coins and choose coins to have a given value. c. Create stories and models, including linear, to illustrate addition and subtraction. d. Select and use appropriate methods to estimate sums and differences or calculate them mentally depending on the context and numbers involved.	**Inquiry:** • What are the various ways numbers can be broken apart and put back together? • What strategies do you use when you want to estimate your answer? • What strategies do use when you add or subtract numbers? • Why do computations create patterns?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the concepts of addition and subtraction. • Students use representations of addition and subtraction to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

3. Demonstration of the understanding of commonly-used fraction concepts.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Partition basic shapes, using common fractions including 1/2, 1/3, and 1/4. b. Partition sets using common fractions such as 1/2, 1/3, 1/4.	**Inquiry:** • How do we represent whole numbers and fractions? • What is the relationship between the numerator and denominator of a fraction? • Why are there fractions?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model fractions. • Students use representations of fractions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

DRAFT

Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

1. Demonstration of place-value concepts to 100 through their relationships to counting, ordering, and grouping.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Count and read numbers to 100 b. Estimate quantities less than 100. c. Organize quantities using place value. d. Locate numbers up to 100 on a number display. e. Compare two sets of objects up to at least 25 using language such as "three more or three fewer".	**Inquiry:** • What do I know about this quantity? How is it related to ten? To a multiple of ten? • How can I count these items in the most efficient and accurate way? • Why did people come up with a place value system?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model quantities using place value concepts • Students use representations of numbers as tens and ones to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

2. Demonstration of the meaning of addition and subtraction, and their relationships.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use addition when putting sets together and subtraction for breaking sets apart or describing the difference between sets. b. Use number relationships including such as doubles, one more or one less, and the relationship between addition and subtraction to solve addition and subtraction problems. c. Identify coins and find the value of a collection of two coins. d. Demonstrate fluency with basic addition and related subtraction facts through sums to 10.	**Inquiry:** • What is addition and what is it used for? • What is subtraction and what is it used for? • What happens to quantities when groups are added together or taken apart? • How do I know I have found all the combinations to make this number? • How are addition and subtraction related?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model the concepts of addition and subtraction. • Students use representations of addition and subtraction to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

3. Recognition of part/whole relationship with common fractions.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Identify unit fractions 1/2, 1/3, and 1/4 as parts of wholes or parts of groups. b. Understand fractions as equal shares or parts.	**Inquiry:** • How can I use manipulatives, drawings, or numerals/symbols to show parts and wholes? • How are the parts and wholes related? • What do fractions tell us?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model wholes and their parts. • Students use representations of wholes and parts to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

1. Representation, comparison, and order of whole numbers less 20.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Count objects by ones to 20 and represent the quantities. b. Identify, read, and write corresponding numerals. c. Compare sets up to 10 objects and use language to describe more, less, or same. d. Identify small groups of objects less than 5 without counting, including zero as "no objects."	**Inquiry:** • What is counting? • How can I make sure my counting is accurate? • How can I compare two quantities? • What are some ways to find a number that is more than another? Less than a number?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model and compare quantities. • Students use equivalent representations of quantities to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

2. Composition and decomposition of whole numbers through 10; join and separate sets with manipulatives or models.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use objects and drawings to model addition and subtraction problems to 10. b. Identify numbers one more or one less than a given number up to 10.	**Inquiry:** • What questions can be answered using addition and/or subtraction? • How can I represent a quantity in a different way? • How many ways can I make a set to match a number? • What happens when two quantities are combined? • What happens when a quantity is separated?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model and compare quantities. • Students use equivalent representations of quantities to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

3. Communication of reasonable estimates in real world number activities.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Identify a set as having more or fewer objects than another set. b. Determine if more or less is needed to change one quantity to another. c. Estimate the number in a set of fewer than twenty objects and refine the estimate while counting.	**Inquiry:** • How can sets and numbers be compared and ordered? • How can I tell if I've made a good guess (estimate)?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model and compare quantities. • Students use estimation to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

1. Counting and grouping objects and numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Match, sort, place in a series, and group objects according to one characteristic. b. Use one-to-on correspondence in counting objects and matching groups of objects. c. Associate a number of objects with names and symbols for numbers.	**Inquiry:** • How can you tell how many? • What do numbers tell us?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model and compare quantities. • Students use equivalent representations of quantities to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Number Sense, Properties, and Operations

Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

2. Use of numbers and counting as a means for solving problems.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Predict and measure quantity. b. Use vocabulary of same, more, less, add, and subtract to express number concept.	**Inquiry:** • How are these groups different? • How can quantities be described? • How many do you think are here?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model and compare quantities. • Students use equivalent representations of quantities to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduate Competencies

The Prepared Graduate Competencies are the Preschool through Grade 12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in Mathematics standards are:

1. Understand the structure of our number system and all of its subsets.
2. Understand quantity through estimation, precision, order of magnitude, and comparison.
3. Fluency with basic symbolic facts and algorithms and selection and use of appropriate methods.
4. Make relative and absolute comparisons between quantities.
5. Recognize and make sense of the ways variability, chance, and randomness appear.
6. Solve problems and make decisions by understanding, explaining, and quantifying variability in data.
7. Understand that equivalence is a foundation of mathematics.
8. Make sound predictions and generations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
9. Apply transformations to numbers, shapes, functional representations, and data.
10. Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.
11. Communicate an effective, logical argument using mathematical justification and proof.
12. Use critical thinking to recognize problematic situations, create a model, and present and defend solutions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

1. Representation of continuous relations and elementary functions (linear, quadratic, absolute value, power, and exponential functions and their inverses) and of discrete relations and elementary functions (arithmetic and geometric sequences) using tables, graphs, symbols, text, and geometric models.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Represent linear, quadratic, absolute value, power, and exponential functions and their inverses in a table, graph, equation, text, and geometric model and convert from one representation to another. b. Demonstrate the relationship between all representations of linear functions using point-slope, slope-intercept, and standard form of a line. c. Categorize sequences as arithmetic, geometric, or neither and develop formulas for the general terms related to arithmetic and geometric sequences	**Inquiry:** • How can you use a table, graph, and function notation to explain how an exponential function is similar to and different from a linear function? • How do you represent the inverse of a function represented in table and graph form? • Why are relations and functions represented in multiple ways?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving transformation of functions. • Students use real world contexts to model a function of best fit. • Students use different representations of functions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of functional thinking, ways of representing functions, and relationships among the elementary functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

2. Analysis of elementary functions and their inverses, by investigating rates of change, intercepts, asymptotes, domain, range, and local and global behavior using all available tools, including technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Given a function in any representation, evaluate a function at a given point in its domain. b. Distinguish between functions and relations defined in any representation. c. Identify intercepts, zeros (or roots), maxima, minima, and intervals of increase and decrease, and asymptotes in a table, graph, and symbolic representation of a function. d. Make qualitative statements about the rate of change of a function, based on its graph or table or values.	**Inquiry:** • Are all functions relations? How can you tell? • What are the key features of elementary functions and how can you find them in a table, graph, and symbolic form? • Why do we classify functions?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving characteristics of functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving the characteristics of functions. • Students use real world contexts to model elementary functions and their inverses. • Students use analysis of elementary functions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of the characteristics of specific functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:
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High School Expectations

Concepts and Skills students know include:

3. Application of transformations, including arithmetic combinations (addition, subtraction, and multiplication) and translations (vertical, horizontal, and dilations) to representations of elementary functions using tables, graphs, symbols, text, and geometric models, using all available tools, including technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Sketch the graph of common elementary functions and translations of those functions. b. Given two functions in any representation, evaluate the sum and difference of the two functions. c. Given two linear or power functions in any representation, evaluate the product of the two functions. d. Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. e. Perform arithmetic combinations on elementary (linear, quadratic, power, and exponential) functions, using all available tools including technology.	**Inquiry:** • What are the effects of performing operations on functions? • Given two functions in graph form, how do you graph the product function?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving characteristics of functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving the characteristics of functions. • Students use real world contexts to model transformations of elementary functions. • Students use application of transformations of elementary functions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of the characteristics of specific functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

4. Understand equivalent forms of expressions, equations, inequalities, and relations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Perform and justify steps in generating equivalent expressions by identifying properties used. b. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology. c. Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions including those involving nth roots.	**Inquiry:** • How do symbolic transformations affect an equation, inequality or expression? • How do you know that two algebraic expressions are equivalent?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving characteristics of functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving the characteristics of functions. • Students use real world contexts to model equivalence forms of expressions, equations, inequalities, and relations. • Students use equivalent forms of expressions, equations, inequalities, and relations to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of the characteristics of specific functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

5. Solutions to equations, inequalities and systems of equations using all available tools, including technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Find solutions to quadratic equations and inequalities (with real roots) by using algebraic methods such as factoring, completing the square, graphing or using the quadratic formula and using calculators, graphing utilities or other technology to solve quadratic equations and inequalities. b. Find solutions to equations involving power and exponential functions; solve these equations graphically or numerically or algebraically using calculators, graphing utilities or other. c. Rewrite literal equations in terms of an unknown variable. d. Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers and from real numbers to complex numbers (particularly when a non-real solution exists). In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients. (From Standard 1) e. Solve systems of linear equations and inequalities with two variables using algebraic methods (substitution and elimination) or graphing.	**Inquiry:** • What makes a solution strategy both efficient and effective? • Are all solutions to an equation valid? • Is the solution reasonable in the context of the problem? • How can we decide when to use an exact answer and when to use an estimate?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving characteristics of functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving the characteristics of functions. • Students use real world contexts to model solutions to equations, inequalities, and systems of equations. • Students use equations, inequalities, and systems of equations to communicate solutions to real world problems.
	• Students build a deep understanding of the characteristics of specific functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

6. Use of elementary functions (linear, quadratic, power, and exponential and their inverses) and their transformations to identify essential quantitative relationships in a situation and to model real-world situations, using all available tools, including technology.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Represent and solve problems in various contexts using linear and quadratic functions. b. Represent and solve problems in various contexts using power and exponential functions, such as investment growth, depreciation and population growth. c. Analyze the reasonableness of a solution in its given context and compare the solution to appropriate graphical and numerical estimates. d. Represent and solve problems involving direct and inverse variations and a combination of direct and inverse variations.	**Inquiry:** • What phenomena can be modeled with particular functions? • Which financial applications can be modeled with exponential functions? Linear function? • How can patterns, relations, and functions be used as tools to describe and explain real-life situations? • What elementary function or functions best represent(s) a given scatter plot of two-variable data?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving characteristics of functions. • Students strategically use technology to explore, organize and communicate mathematical concepts involving the characteristics of functions. • Students use real world contexts to model elementary functions and their inverses. • Students use analysis of elementary functions to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of the characteristics of specific functions. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving the characteristics of functions.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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Eighth Expectations

Concepts and Skills students know include:

1. Descriptions of linear pattern, relation, and function relationships using equations, tables, and graphs in problem solving situations, and conversions among these representations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Convert from one representation of a linear function to another, including situations, tables, equations (slope-intercept form), and graphs. b. Use representations of linear functions to analyze situations and solve problems.	**Inquiry:** • How can different representations of linear patterns present different perspectives of situations? • How can a relationship be analyzed with tables, graphs and equations? • Why are functions represented in multiple ways?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model linear patterns, relations, and functions. • Students use different representations of linear patterns to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

2. Use of the properties of algebra, equality, and inequality to create equivalent algebraic expressions and solve linear equations and inequalities using a variety of methods, including technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use the distributive, associative, and commutative properties to simplify algebraic expressions. b. Solve one variable equations, including those involving multiple steps, rational numbers, variables on both sides, and the distributive property. c. Solve inequalities in one variable (including negative coefficients) and graph the solution on a number line. d. Represent the distributive property in a variety of ways including numerically, geometrically, and algebraically.	**Inquiry:** • How can equivalence exist with algebraic expressions? • How do you know if an algebraic algorithm is sensible? • How can we change an algebraic equation without changing the value? • If we change the way an equation or expression looks, do we change its value?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model properties of algebra, algebraic expressions, equations, and inequalities. • Students use properties of algebra, algebraic expressions, equations, and inequalities to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

3. Application of properties of linear relations and functions to interpret linear situations using algebraic methods, graphs, and technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Identify and interpret the slope and y-intercept in graphs, in tables, and from equations in slope-intercept form. b. Calculate the slope between two points on the coordinate plane or the rate of change between two sets of values in a table. c. Model and graph two linear equations in slope-intercept form on the same coordinate plane and interpret the point of intersection.	**Inquiry:** • How can we use mathematics to describe change? • How can change best be represented mathematically? • Why are patterns and relationships represented in multiple ways?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model properties linear relations and functions. • Students use properties of linear relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

1. Description of linear patterns, relations and functional relationships using equations, tables, and graphs in problem solving situations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Given a linear situation, identify variables and write an equation. b. Given a linear equation, substitute input values to create a table and graph coordinate points in all four quadrants.	**Inquiry:** • What makes a pattern linear? How can you identify linear relationships from different representations? • Why are linear patterns important to know?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model linear patterns, relations, and functions. • Students use different representations of linear patterns to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra,

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Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

1. Use of a variable as a representation for unknown quantities.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Describe patterns by using words and by using variables with mathematical symbols. b. Evaluate expressions by substituting whole number values for variables.	**Inquiry:** • What purposes do variable expressions serve? • How can patterns be represented verbally and by variable expressions?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model variables. • Students use patterns and variables to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra.

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Content Area: Mathematics
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Sixth Grade Expectations

Concepts and Skills students know include:

2. Analysis and description of patterns, with and without variables, using words, tables, and graphs.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Extend and describe the rule for arithmetic and geometric sequences. b. Model linear situations using tables and graphs, and convert among these two representations. c. Given a linear equation, substitute non-negative input values to create a table and graph coordinate points in the first quadrant.	**Inquiry:** • How are tables and graphs of similar data related? • How do you extend patterns? • What is a rule? How does a rule help?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and algebra. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and algebra. • Students use real world contexts to model patterns. • Students use different representations of linear patterns to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving patterns and algebra.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

1. Description, display, and analysis of patterns and relationships using a variety of tools including words, tables, graphs and technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Analyze and describe patterns and relationships using words, tables, graphs, and technology. b. Recognize, explain, and extend linear and non-linear patterns and relationships in solving problems. c. Represent, describe, and explain patterns and relationships including variables.	**Inquiry:** • How can a relationship be described or represented? • How does an "unknown" change the relationship? • How can patterns and relationships be used as tools to describe and explain real-life situations? Which is best?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

2. Analysis of relationships to identify how a change in one number results in a change in another number using all four operations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Express change relationships with if/then statements, input/output boxes, function tables, and rule statements. b. Select, describe and use symbols to express unknown quantities. c. Use patterns to solve problems.	**Inquiry:** • How does changing one number in a relationship result in changing another number? • How do you know when you've found the rule for a pattern? • How does knowing a rule for a pattern help you solve problems?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts patterns and relationships. • Students use real world contexts to model change relationships. • Students use different representations of change relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

1. Recognition, reproduction, extension, creation, or description of patterns and sequences that are formed using a variety of materials including manipulatives and numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Skip count using numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 from zero and from randomly selected starting places. b. Reproduce, extend, create, and describe patterns using pictures, geometric shapes, and numbers.	**Inquiry:** • What patterns do you notice when you skip count? • How do patterns help you skip count from randomly selected places? • How do patterns help you make predictions? • How do we show patterns?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

2. Demonstration of understanding of patterns and relationships by solving for missing elements or unknown quantities in problems and simple equations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use number relationships to find the missing number in a set. b. Use a symbol to represent and find an unknown quantity in a problem situation.	**Inquiry:** • How do you know the next term in a pattern? • Why do we use symbols to represent missing numbers in a pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

3. Observation and explanation about how a change in one quantity can produce a change in another, given a relationship between two quantities.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Write the rule for a scenario that is increasing or decreasing. b. Apply a given rule to fill in missing elements.	**Inquiry:** • How can a change in one number result in the change of another? • How can knowing a rule help you solve pattern problems? • How do you know when you are applying the correct rule to a pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
Standard: Patterns, Functions, and Algebraic Structures

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

1. Identification, description, and extension of patterns formed using a variety of materials and manipulatives.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Analyze patterns in multiples. b. Skip count by 25, 50, and 100. c. Use known multiplication facts to solve unknown multiplication problems. d. Extend pattern sets with pictures, charts, and open sentences.	**Inquiry:** • How do you know when there is a pattern? • What patterns do you notice when you count by 25, 50, and 100? • How does knowing a multiplication fact help you find answers to other problems?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

2. Recognition, understanding, and use of patterns to solve problems

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Find missing elements in a pattern. b. Complete input/output tables and guess my rule statements. c. Solve problems of elapsed time.	**Inquiry:** • How are patterns useful? • How can knowing a rule help you solve pattern problems? • How do you know when you are applying the correct rule to a pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns. • Students use different representations of patterns to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

1. Identification of patterns and relationships and use of that information to solve a problem using a variety of materials.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Use ten based strategies to solve addition and subtraction facts to 20. b. Demonstrate the structure of numbers as tens and ones in addition and subtraction. c. Communicate the relationship between addition and subtraction, and use this relationship to efficiently solve problems.	**Inquiry:** • How is ten a helpful number? • How are addition and subtraction related? • How can you use addition to help you find a solution to a subtraction problem?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

2. Recognition, description, reproduction, extension, and creation of increasingly complex patterns using a variety of materials including numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Count objects by groups using 2, 5, and 10 both verbally and written. b. Identify a missing number in a sequence, and describe a rule. c. Create a three to five element pattern using a variety of materials such as numbers, letters, shapes, and manipulatives. d. Extend a three to five element pattern using a variety of materials such as numbers, letters, shapes, and manipulatives.	**Inquiry:** • What patterns do you notice when you count by 2, 5, and 10? • How can you use a pattern to help you solve a problem? • What patterns can you create? • How does finding patterns help in counting?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

1. Creation and extension of a patterns based on rules.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Count objects by groups of 2 or 5. b. Create a pattern based on a rule. c. Extend a pattern based on a rule.	**Inquiry:** • What patterns do you notice? • How can you extend this pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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First Grade Expectations

Concepts and Skills students know include:

2. Recognition of patterns and usage of that information to solve a problem.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Describe how a pattern repeats or grows and extend the pattern. b. Use known number relationships such as doubles plus or minus one to solve problems.	**Inquiry:** • What pattern do you notice? • What is the repeating unit of the pattern? • How can you use that pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

1. Recognition, description, reproduction, extension, and creation of simple patterns using a variety of materials including numbers.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Create a two element pattern using a variety of materials such as numbers, letters, shapes, and manipulatives. b. Extend a two element pattern using a variety of materials such as numbers, letters, shapes, and manipulatives.	**Inquiry:** • What is a pattern? • Where are patterns found? • How do you know there is a pattern?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

2. Description of basic relationships with numbers to 10.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Generalize the counting sequence pattern from counting all to knowing "one more" and "one less". b. Communicate the relationship between composing and decomposing numbers.	**Inquiry:** • How many ways can a set be broken apart or put together?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Content Area: Mathematics
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Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

1. Recognition and creation of patterns.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Group a number of similar objects into simple categories.	**Inquiry:** • What are the ways to group objects?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving patterns and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving patterns and relationships. • Students use real world contexts to model patterns and relationships. • Students use different representations of patterns and relationships are used to communicate solutions to real world problems.
	Nature of Mathematics: • Students recognize and represent mathematical relationships and analyze change. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving pattern and relationships.

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Data Analysis, Statistics, and Probability

Data and probability sense provides a student with tools to make meaning of information and uncertainty. Being a student of mathematics involves asking questions and then gathering or using data to answer these questions. Students learn that there are a variety of data analysis and statistics strategies that enable them to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduate Competencies

The Prepared Graduate Competencies are the Preschool through Grade 12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in Mathematics standards are:

1. Understand the structure of our number system and all of its subsets.
2. Understand quantity through estimation, precision, order of magnitude, and comparison.
3. Fluency with basic symbolic facts and algorithms and selection and use of appropriate methods.
4. Make relative and absolute comparisons between quantities.
5. Recognize and make sense of the ways variability, chance, and randomness appear.
6. Solve problems and make decisions by understanding, explaining, and quantifying variability in data.
7. Understand that equivalence is a foundation of mathematics.
8. Make sound predictions and generations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
9. Apply transformations to numbers, shapes, functional representations, and data.
10. Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.
11. Communicate an effective, logical argument using mathematical justification and proof.
12. Use critical thinking to recognize problematic situations, create a model, and present and defend solutions.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

1. Design of censuses, surveys, observational and experimental studies to answer statistical questions and understand the types of inferences can legitimately be drawn from each.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Formulate appropriate research questions that can answered with statistical analysis. b. Determine appropriate data collection methods to answer a research question. c. Explain how data will be analyzed to provide answers to a research question.	**Inquiry:** • What types of questions and data collection methods are appropriate for statistical analysis? • How do you reduce bias in question design and data collection methods? • How can the results of a statistical investigation be used to support an argument?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving statistical analysis. • Students strategically use technology to explore, organize and communicate mathematical concepts involving statistical analysis. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

2. Evaluation of the quality of observational studies, surveys, and experimental studies.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Identify the characteristics of a well-designed and well-conducted survey. b. Differentiate between the value of observational studies as useful for suggesting patterns in data and relationships between variables and experimental studies as useful for establishing cause and effect.	**Inquiry:** • What is the difference between an observational study and an experiment? • What role does randomization play in collecting data in a study? • How do we gauge the quality of a study? • How does sampling build or erode our confidence in the claims we can make?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving statistical analysis. • Students strategically use technology to explore, organize and communicate mathematical concepts involving statistical analysis. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:
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High School Expectations

Concepts and Skills students know include:
3. Selection of appropriate methods to collect, organize and analyze data (numerical and categorical, univariate and bivariate) using tables, graphical displays, and numerical summary statistics.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Identify and choose appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics. b. Define and explain how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics. c. Recognize association between two categorical variables. d. Recognize when the relationship between two numerical variables is reasonably linear, use Pearson's correlation coefficient as a measure of strength, and apply the least-squares criterion for line fitting.	**Inquiry:** • Which display(s) should be utilized in a given statistical analysis? • What happens to sample-to-sample variability when you increase the sample size? • What is the association between two categorical variables? • How do you determine an outlier? • When should we sample? When is sampling better than using a census? • What size sample is sufficient? **Applying Mathematics in Society and Using Technology:** • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving statistical analysis. • Students strategically use technology to explore, organize and communicate mathematical concepts involving statistical analysis. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems. **Nature of Mathematics:** • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

4. Interpretation of results of a study, including inferences and predictions.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Define and explain the meaning of significance (both practical and statistical). b. Explain the role of p-values in determining statistical significance. c. Determine the margin of error associated with an estimate of a population characteristic.	**Inquiry:** • What is the practical significance of a given study and how does this differ from statistical significance? Why is it important to know the difference? • How can you make generalizations from the study? • What does margin of error mean?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving statistical analysis. • Students strategically use technology to explore, organize and communicate mathematical concepts involving statistical analysis. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

5. Experimentation with random phenomena using probability and simulation with technology.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Develop simulations that demonstrate probability as a long-run relative frequency b. Apply and solve problems using the concept of independence and conditional probability. c. Apply and solve problems using the concept of mutually exclusive properties when combining probabilities. d. Evaluate and interpret probabilities using a normal distribution with tables and technology. e. Find the mean (expected value) and standard deviation of a discrete random variable *X*. f. Interpret the expected value and standard deviation of a discrete and continuous random variable *X*. g. Apply combinatorics to determine the theoretical probability of a success in a real-world situation.	**Inquiry:** • How can you tell if phenomena are random? • How and why do you use simulations to determine probability when the theoretical probability is unknown? **Applying Mathematics in Society and Using Technology:** • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving statistical analysis. • Students strategically use technology to explore, organize and communicate mathematical concepts involving statistical analysis. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems. **Nature of Mathematics:** • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

1. Analysis and interpretation of two-variable data.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Given a scatterplot, calculate quadrant count ratio to quantify the magnitude and strength of the association between two variables for numeric data as positive, negative, or no correlation. b. Given a scatterplot suggesting a linear relationship, draw a line of fit to make predictions. c. Use time series plots (line graphs) to analyze the trend of a set of data over time.	**Inquiry:** • How can different quantities be expressed depending on the desired degree of precision? • How are fractions, decimals and percents used to make comparisons? • How does one decide whether to express a quantity as a fraction, decimal or percent?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving number. • Students strategically use technology to explore, organize and communicate mathematical concepts involving numbers. • Students use real world contexts to model statistical analysis. • Students use statistical analysis to communicate solutions to real world problems.
	Nature of Mathematics: • Students build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving numbers.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

2. Recognition of the ways in which statistics can be misused.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Recognize the misuse of statistics from examples in newspapers, magazines and other popular media. b. Recognize the use of deceptive scales on a graph that make differences look much larger than they are, or the use of pictographs with areas that are proportioned incorrectly. c. Identify how results of a survey could predict incorrect results based on incorrect sampling techniques or biases in sample selection, or question design.	**Inquiry:** • How can you determine the reliability of data and its source? • What are common ways statistics are misused or misinterpreted? • How does an inaccurate graph affect the representation of data? • How does a sample affect confidence in results? • What negative consequences in society could arise if inferences are based on misleading statistics?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model misuse of statistics. • Students use recognition of the misuse of statistics to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

1. Analysis and interpretation of one-variable data regarding shape, center, and spread.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Distinguish between median as middle number and mean as balance point for an ordered set of data. b. Use Mean Average Deviation (MAD) to analyze the spread of a set of data. c. Construct and interpret dot plots, histograms, and stem-and-leaf plots. d. Construct and interpret a box plot using the five number summary and identify the interquartile range (IQR) for a set of data. e. Compare sets of data using shape (skewed, normal, uniform), with appropriate measures of center (mean, median, mode), and appropriate measures of spread (range, IQR, MAD).	**Inquiry:** • What are the methods used to describe data? • Why are measures of data important? • How does data distribution affect measures of central tendency?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model analysis and interpretation of one-variable data. • Students use analysis and interpretation of one-variable data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

2. Analysis of data using concepts of relative frequency.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Given a frequency table, calculate relative frequencies. b. Given a set of categorical data, construct a circle graph. c. Recognize experimental probability as a relative frequency (percent).	**Inquiry:** • What is the value of using relative frequency to analyze data? • What is experimental probability?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model relative frequency. • Students use analysis of relative frequency to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

1. Formulation of questions and collection of data.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Formulate questions for populations larger than the classroom. b. Recognize that a sample may not represent a population accurately. c. Recognize bias in surveys. d. Utilize techniques to determine a random sample.	**Inquiry:** • What makes a good question for a survey? • How does a sampling design reduce bias? • How might the sample for a survey affect the results of the survey? • What makes a sample random? • When and why would you question the results of a survey? • What are the sources, types and effects of bias?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model collection of data. • Students use formulation of questions and collection of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

2. Employment of counting strategies to determine sample space.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Make a table to determine possible outcomes of two compound events. b. Make a tree diagram to determine possible outcomes of three compound events. c. Make an organized list to find combinations or permutations.	**Inquiry:** • How can you determine the possible outcomes of a compound event? • What are the ways to organize the possible outcomes for an event? • Why is it important to consider all of the possible outcomes of an event?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model sample spaces. • Students use samples spaces to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

3. Exploration of the concept of chance using experimental and theoretical probability.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Determine probabilities through experiments or simulations. b. Express the probability of an event using fractions, decimals, and percents. c. Predict outcomes of events using experimental and theoretical probabilities.	**Inquiry:** • How can you determine and gauge if an event is likely or not? • How are predictions made based on the outcomes of a probability experiment? • How is probability described?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data, statistics, and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data, statistics, and probability. • Students use real world contexts to model experimental and theoretical probability. • Students use exploration of experimental and theoretical probability to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data, statistics, and probability

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

1. Formulation of questions, collection, construction, analysis, and interpretation of data displays to solve problems or communicate results.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Formulate a question and hypothesis to design appropriate data collection and display methods. b. Select and create displays of data including double bar graphs, time plots, or circle graphs. c. Interpret data using the concepts of shape of distribution, range, mode, median and mean. d. Draw conclusions, and make convincing arguments based on categorical and numerical data analysis.	**Inquiry:** • How can the collection, organization, interpretation and display of data be used to answer questions? • What conclusions can be made from data?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

2. Generation and analysis of data from chance devices and events.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Apply the concepts of degrees of likelihood using fractions, decimals, and percentages. b. Demonstrate an understanding that larger number of trials give more accurate data. c. Design a game involving a chance device such as number cube or spinner and explain why the game is fair or unfair.	**Inquiry:** • How can experimenting with chance devices be used to make predictions or draw conclusions? • What makes a game fair?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to generate and analyze data. • Students use generation and analysis data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

1. Formulation of questions, collection, construction, analysis, and interpretation of data displays to solve problems or communicate results.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Compose questions to clarify purpose and generate data. b. Determine the most appropriate display for the purpose. c. Collect data from class experiments or multi-classroom surveys. d. Interpret data using the concepts of shape of the distribution, range, mode and median. e. Determine if data gathered is impacted by error or reflects variability.	**Inquiry:** • What makes a data representation useful? • How do you choose the best representation for data? • What affects the quality of data?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

2. Solutions to problems involving combinations

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Organize all possible combinations in a list or chart. b. Use estimation strategies to determine reasonable answers.	**Inquiry:** • How can you know all of the possible combinations for an event? • How can knowing the possible outcomes for an event help you make predictions?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model combinations. • Students use combinations to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

3. Generation, analysis, and representation of possibilities.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Conduct experiments using chance devices, and make predictions with support using the concepts of impossible, unlikely, likely, and certain. b. Formulate questions, conduct surveys and make predictions using the concepts of impossible, unlikely, likely and certain. c. Represent possibilities from chance devices with fractions or percentages.	**Inquiry:** • How can you tell how likely an event is? • How do we communicate the likelihood of an event? • What is the difference between likely and certain events?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model simple probability. • Students use simple probability to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

1. Collection, construction, analysis, and interpretation of data displays to solve problems or communicate results from experiments or surveys.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Create pictographs, bar graphs, line graphs, dot plots, and frequency tables from a data set. b. Interpret data using the concepts of range median, mode, clusters and gaps. c. Collect and organize data from simple experiments or surveys in class. d. Pose questions to generate data. e. Determine which display form is a most appropriate to the data.	**Inquiry:** • What are the ways data can be displayed? • How do data displays help us understand information? • What can data tell you about the people you survey? • What makes a good survey question?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

2. Generation of data and predictions using chance devices.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Investigate chance devices such as coins, spinners, and dice. b. Apply the concepts of impossible, unlikely and likely. c. Determine if a chance device is fair or unfair.	**Inquiry:** • How can you tell how likely an event is? • How do we communicate the likelihood of an event? • What is fair? • How would you know a game is unfair?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model simple probability. • Students use simple probability to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

1. Collection, generation, reading, analysis, and interpretation of displays of data.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Construct charts, picture graphs, bar graphs, and tables from a data set. b. Sort and classify object and information with two attributes or variables. c. Interpret data using concepts of largest, smallest, most often, and middle. d. Compare individual data to group data. e. Determine questions which will generate variable data.	**Inquiry:** • What are the ways data can be displayed? • How do data displays help us understand information? • What can data tell you about the people you survey? • What makes a good survey question?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

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Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

2. Generation of data and predictions using chance devices

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Collect data using chance devices. b. Apply the concepts of likely or not likely based on data collection.	**Inquiry:** • How can you tell how likely an event is? • How do we communicate the likelihood of an event?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model simple probability. • Students use simple probability to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

1. Generation, reading, analysis, and interpretation of displays of data.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Construct displays of classroom data including charts, picture graphs, and bar graphs from a data set. b. Read and explain information in charts, picture graphs, bar graphs and tables. c. Interpret data by applying the concepts of largest, smallest and most often. d. Recognize when a question will generate variable data.	**Inquiry:** • What are the ways data can be displayed? • How do data displays help us understand information? • What kinds of questions generate data? • What questions can be answered by a data representation?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

1. Categorization and classification of objects and information

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Sort objects and explain categorization. b. Make combinations by matching items from one group to another group.	**Inquiry:** • What are the ways to group objects?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to categorize and group objects and information. • Students use categorization and classification to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

2. Description and comparison of data displays.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Describe charts, picture graphs, and bar graphs. b. Collect classroom data c. Identify and compare own data to group's data.	**Inquiry:** • What are the ways data can be displayed? • How do data displays help us understand information?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving data and probability. • Students strategically use technology to explore, organize and communicate mathematical concepts involving data and probability. • Students use real world contexts to model data collection and displays. • Students use different representations of data to communicate solutions to real world problems.
	Nature of Mathematics: • Students ask questions, gather and use data to make sense of information and uncertainty. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving data collection.

DRAFT

Content Area: Mathematics
Standard: Data Analysis, Statistics, and Probability

Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a.	**Inquiry:** •
	Applying Mathematics in Society and Using Technology: •
	Nature of Mathematics: •

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Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Being a student of mathematics involves analyzing the characteristics and relationships of shapes and structures, engaging in logical reasoning, and using tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real-world and throughout mathematics.

Prepared Graduate Competencies

The Prepared Graduate Competencies are the Preschool through Grade 12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in Mathematics standards are:

1. Understand the structure of our number system and all of its subsets.
2. Understand quantity through estimation, precision, order of magnitude, and comparison.
3. Fluency with basic symbolic facts and algorithms and selection and use of appropriate methods.
4. Make relative and absolute comparisons between quantities.
5. Recognize and make sense of the ways variability, chance, and randomness appear.
6. Solve problems and make decisions by understanding, explaining, and quantifying variability in data.
7. Understand that equivalence is a foundation of mathematics.
8. Make sound predictions and generations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
9. Apply transformations to numbers, shapes, functional representations, and data.
10. Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.
11. Communicate an effective, logical argument using mathematical justification and proof.
12. Use critical thinking to recognize problematic situations, create a model, and present and defend solutions.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

High School Expectations

Concepts and Skills students know include:

1. Determination and utilization of the area of irregular shapes, and surface area and volume of cones and pyramids, cylinders and prisms, and spheres.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Calculate (or estimate when appropriate) through successive iterations the perimeter and area of a two-dimensional irregular shape. b. Justify, interpret, and apply the use of formulas for the area, surface area, and volume of cones/pyramids, spheres, and cylinders/prisms. c. Solve for unknown quantities of two-dimensional shapes involving area and perimeter and with three-dimensional shapes involving volume and surface area. d. Apply the effect of dimensional change of length, area, and volume and utilize appropriate units and scales for problem solving situations involving perimeter, area, and volume. e. Analyze real-world situations involving perimeter and area of irregular shapes and volume of cones/pyramids, cylinders/prisms, and spheres. f. Develop and justify conjectures about relationships among properties of shapes in two- and three-dimensions using construction tools, including technology.	**Inquiry:** • When the surface area and volume changes, how does an aspect of dimension change? • What is the relationship between a line, a two-dimensional shape, and a three-dimensional shape? • How might surface area and volume be used to explain biological differences in animals? • How do you measure the area of an irregular shape? • How can surface area be minimized while maximizing volume? **Applying Mathematics in Society and Using Technology:** • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model perimeter, area, and volume. • Students use different representations of perimeter, area, and volume to communicate solutions to real world problems. **Nature of Mathematics:** • Students analyze the characteristics and relationships of shapes and structures; engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

2. Relationships among two- and three-dimensional geometric figures, including congruence, similarity and symmetry.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Classify polygons according to their similarities and/or differences. b. Solve for unknown attributes of geometric shapes based on their congruence, similarity, or symmetry. c. Know and apply properties of angles including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve problems and logically justify results. d. Develop and justify conjectures and solve problems about geometric figures including definitions and properties (congruence, similarity, and symmetry). e. Design a geometric structure with accurate and appropriate units of measure. f. Develop and justify conjectures about relationships among properties of shapes in two dimensions (polygons and circles) and three dimensions (cones and pyramids, cylinders and prisms, and spheres) using construction tools, including technology.	**Inquiry:** • How can you prove that two figures are congruent? • What is the sum of the exterior angles of a polygon? • What makes a good definition of a shape?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model relationships among geometric figures. • Students use relationships among geometric figures to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

DRAFT

Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:
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High School Expectations

Concepts and Skills students know include:
3. Utilization of Cartesian coordinate system in geometric contexts.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Determine the midpoint of a line and the distance between two points in the Cartesian coordinate plane. b. Investigate conjectures and problems involving two-dimensional objects represented with Cartesian coordinates. c. Represent transformations (reflection about the *x*- and *y*-axes, translation, rotation about the origin, and dilations) using the Cartesian coordinates. d. Develop arguments to establish what remains invariant and what changes after a transformation (reflection about the *x*- and *y*-axes, translation, rotation about the origin, and dilations) using Cartesian coordinates. e. Using construction tools, including technology, make and/or defend conjectures about relationships among properties of shapes in the coordinate plane. f. Develop and justify conjectures about relationships among properties of shapes including those formed through transformation in the coordinate plane using construction tools, including technology.	**Inquiry:** • What advantage, if any, is there to using the Cartesian coordinate system for proving properties of shapes? • What does it mean for two lines to be parallel? How can you physically verify that two lines are really parallel? • What happens to the coordinates of the vertices of shapes when you apply different transformation in the plane? **Applying Mathematics in Society and Using Technology:** • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model Cartesian coordinate geometry. • Students use geometry on the Cartesian coordinate system to communicate solutions to real world problems. **Nature of Mathematics:** • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

4. Representation and application of simple transformations of objects in the plane including translations, reflections, rotations and dilations.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Develop arguments to establish what remains invariant and what changes after a transformation. b. Develop conjectures about properties of geometric figures, using construction tools, including technology. c. Develop and justify conjectures about properties of geometric figures using two-column proofs, paragraph proofs, flow charts, and/or illustrations.	**Inquiry:** • What remains invariant and what changes when a shape is flipped? • How do you know that a shape tessellates the plane?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model simple transformations. • Students use representation and application of simple transformations to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:
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High School Expectations

Concepts and Skills students know include:
5. Validation of geometric conjectures using deduction, proving theorems and reasonable justifications.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Explain the roles of axioms, definitions, undefined terms, and theorems in logical argument and formal proofs. b. Recognize the logical relationships between an if-then statement and its inverse, converse, and contrapositive. c. Construct logical arguments using two-column proofs, paragraph proofs, flow charts, or illustrations. d. Assess the validity of a logical argument and provide a counter example to disprove a statement.	**Inquiry:** • What makes a proof appropriate? • How do you know when you've proven something? • What does it take to verify a conjecture? • How does a geometric proof differ from proof in other contexts? • When is the converse of an if-then statement true? • How does logic help you prove something?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model validation of geometric conjectures. • Students use geometric conjectures to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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High School Expectations

Concepts and Skills students know include:

6. Indirect measurement of quantities using techniques of algebra or geometry.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Apply right triangle trigonometry to find indirect measures of lengths and angles. b. Apply the laws of sines and cosines to find indirect measures of lengths and angles. c. Apply the Pythagorean theorem, sine, cosine and tangent functions to analyze and propose a solution to a real-world situation.	**Inquiry:** • How can you measure something that you can't measure physically? • How can you make a corner square?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving perimeter, area, and volume. • Students strategically use technology to explore, organize and communicate mathematical concepts involving perimeter, area, and volume. • Students use real world contexts to model measurement. • Students use indirect measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

1. Description, analysis and reasoning about angles, parallel and perpendicular lines, and distance on a coordinate plane.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Classify quadrilaterals and apply angle and side properties, including the sum of the interior angles. b. Apply properties of complimentary, supplementary, and vertical angle relationships. c. Apply properties of parallel lines including corresponding angles and alternate interior angles.	**Inquiry:** • How can geometric relationships among lines and angles be generalized, described, and quantified? • How are squares and rhombuses like and unlike each other? • What relationships exist between quadrilaterals? • How do line relationships affect angle relationships?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model relationships among and within angles and lines. • Students use analysis and reasoning about angles and lines to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Eighth Grade Expectations

Concepts and Skills students know include:

2. Estimation, creation, and usage of direct and indirect measurements to describe and make comparisons.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use properties of similar triangles to find unknown lengths. b. Use the Pythagorean Theorem to find unknown lengths in right triangles. c. Given a situation, draw a picture and use similar triangles to find unknown lengths. d. Use proportions to estimate length, distance, capacity, and weight.	**Inquiry:** • How can you measure something that is inaccessible? • What is proportional reasoning and how is it used in geometry and measurement?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model indirect measurement. • Students use estimation and indirect measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

1. Estimation and calculation of surface area, volume, circumference of circle, and area of a circle.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Calculate the surface area and volume of right cylinders and right prisms. b. Calculate area of regular polygons, circumference and area of circles, and area of composite figures. c. Identify and construct two-dimensional nets of prisms and cylinders.	**Inquiry:** • How are surface area and volume like and unlike each other? • What do surface area and volume tell you about an object? • How are one, two, and three dimensional units of measure related?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model surface area and volume. • Students use surface area and volume to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Seventh Grade Expectations

Concepts and Skills students know include:

2. Application of ratio, proportion, and similarity in problem-solving situations

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Describe the relationship between the circumference, diameter, and area of a circle. b. Read and interpret scales on maps. c. Use proportions to convert from one set of units to another within customary and metric systems.	**Inquiry:** • How are circumference and diameter related? • In what ways can proportion help us solve problems? • How do you know when a proportional relationship exists?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model proportion and similarity. • Students use proportion and similarity to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

1. Identification, comparison, conjecturing, and reasoning about properties of two-dimensional shapes.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Develop and apply formulas and procedures for finding area of triangles, parallelograms, and, trapezoids b. Describe properties of two-dimensional shapes with accurate vocabulary and notation. c. Classify triangles and apply angle and side properties, including the sum of the interior angles.	**Inquiry:** • What are the ways to find areas of figures? • What does area tell you about a figure? • What properties affect the area of figures?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model properties of two-dimensional shapes. • Students use properties of two-dimensional shapes to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

2. Usage of equivalent measurements for length, capacity, and weight.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Connect metric prefixes to place value. b. Solve simple conversion problems within customary and metric systems.	**Inquiry:** • How does the metric system relate to our number system? • Why does it help to be able to convert within measurement systems?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model equivalent measurements. • Students use equivalence of measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Sixth Grade Expectations

Concepts and Skills students know include:

3. Solve problems that require attention to both approximation and precision of measurement.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Measure to the nearest millimeter and sixteenth of an inch. b. Select and use appropriate units and tools to measure to the degree of accuracy required in a problem solving situation. c. Use protractor to measure angles to the nearest degree.	**Inquiry:** • What does it mean to approximate measurement? • When does precision in measurement matter and why?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving shape and dimension. • Students strategically use technology to explore, organize and communicate mathematical concepts involving shape and dimension. • Students use real world contexts to model approximate and precise measurement. • Students use approximate and precise measurements to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving shape and dimension.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:
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Fifth Grade Expectations

Concepts and Skills students know include:
1. Identification, description, and analysis of spatial relationships and properties of two and three-dimensional figures.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Relate two-dimensional shapes to three-dimensional shapes using faces, edges, and vertices. b. Predict and describe the results of transformations: translations, reflections, rotations. c. Identify and measure angles. d. Apply concepts of parallel, perpendicular, and congruence and line symmetry.	**Inquiry:** • How can spatial relationships be described by use of specific geometric language? • How do geometric relationships help us solve problems? • What changes and what stays the same when objects are rotated, reflected, or translated? • For any given three-dimensional object, what is the relationship between faces, edges, and vertices?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Fifth Grade Expectations

Concepts and Skills students know include:

2. Selection and usage of appropriate tools and units for estimating and measuring to solve problems.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Accurately measure length to the nearest 1/8 inch or millimeter. b. Determine the perimeter of polygons and area of rectangles. c. Distinguish between appropriate units for area and linear measures. d. Model volume using cubic units. e. Use, apply, and select appropriate scales on number lines, graphs, and maps.	**Inquiry:** • How can measurements be used to solve problems involving shapes? • What are the ways we can describe the size of an object or shape? • How does what we measure influence how we measure?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

1. Comparison and classification of geometric figures.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Classify geometric figures as polygons, circles, types of triangles, and types of quadrilaterals. b. Compare geometric figures according to the attributes of congruence, symmetry, and angle types.	**Inquiry:** • What are the ways to compare and classify geometric figures? • Why do we classify shapes?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

2. Solutions to problems using geometric relationships, transformations, and spatial reasoning.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Identify parallel, perpendicular, and intersecting line segments. b. Construct designs using transformations (reflections, translations, and rotations). c. Name and locate points specified by ordered number pairs on a coordinate grid.	**Inquiry:** • How can transformations be used to make designs? • How can you describe location of an object on a coordinate grid? • What are some ways line segments can be related?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Fourth Grade Expectations

Concepts and Skills students know include:

3. Selection and usage of appropriate tools and units for estimating and measuring to solve problems.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Accurately measure length to the nearest 1/4 inch or 1/2 cm. b. Model area using square units. c. Estimate and measure length, time, and weight. d. Distinguish between area and perimeter. e. Convert using equivalencies within standard and within metric measuring systems	**Inquiry:** • How do you decide how close is close enough? • How can you describe the size of geometric figures? • Why do we need standard units of measure?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

1. Recognition of basic attributes and properties of geometric figures and usage of these to describe, analyze, and classify shapes.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Construct and describe two-dimensional shapes by attributes and properties such as sides, angles, and symmetry. b. Recognize and demonstrate transformations (reflections, translations, and rotations) of basic shapes or designs. c. Use geometric properties of points and line segments to describe figures.	**Inquiry:** • How can you describe geometric figures?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Third Grade Expectations

Concepts and Skills students know include:

2. Selection and usage of appropriate tools and units for time, weight, temperature, and precise linear measurements.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Use standard units to measure to the nearest 1/2 or whole inch or centimeter. b. Estimate and measure distance and perimeter. c. Identify common units of time, weight, and temperature and their appropriate use.	**Inquiry:** • How important is precise measurement? How close is close enough? • Why are there different units to measure different things?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement to communicate solutions to real world problems.
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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

1. Application of spatial relationships and properties to describe and transform geometric figures.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Recognize, describe, and build geometric figures using properties such as shape, size, and position. b. Identify symmetry in two-dimensional figures. c. Recognize measurable attributes to describe and estimate size of objects.	**Inquiry:** • How can we describe geometric figures?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Second Grade Expectations

Concepts and Skills students know include:

2. Selection and usage of appropriate tools and units for measurements using non-standard and standard measures.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Identify the measurable attribute and appropriate unit of measure b. Use common objects as non-standard units. c. Use standard linear measuring tools to measure to the nearest whole unit.	**Inquiry:** • What are the different things we can measure? • Why are there standard measurement units?
	Applying Mathematics in Society and Using Technology: • Students justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

1. Usage of part-whole relationships, attributes, and properties to compose and decompose shapes.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Recognize, describe, and construct shapes according to given relationships, attributes, or properties. b. Sort geometric figures by shape and size and explain reasoning. c. Combine and take apart shapes and describe results.	**Inquiry:** • How can we describe geometric figures?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

2. Recognition and description of shapes.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Describe the attributes of common shapes. b. Compare everyday objects to common shapes.	**Inquiry:** • How can you describe shapes? • What shapes do you see around you?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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First Grade Expectations

Concepts and Skills students know include:

3. Comparison and ordering of objects and events according to measurable attributes.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Measure the length of common objects using nonstandard units. b. Distinguish units of time (day, night, morning, afternoon, hours) and connect them to common events. c. Compare objects by length and weight.	**Inquiry:** • How can you tell when one thing is bigger than another?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement is used to communicate solutions to real world problems.
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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

1. Recognition, naming, description and construction of basic shapes.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Identify, build and draw common shapes such as square, rectangle, triangle, and circle. b. Demonstrate, describe, and classify shapes by their attributes c. Identify positions of objects in space.	**Inquiry:** • How are shapes like each other? How are they different? • What are ways to describe where an object is?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships are used to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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Kindergarten Expectations

Concepts and Skills students know include:

2. Comparison and ordering of objects according to measurable attributes.

Evidence Outcomes Students can:	21st Century Skills and Readiness Competencies
a. Recognize and compare attributes of length height, weight, capacity of objects. b. Use estimates of measurements from everyday experiences. c. Order several objects by length, height, weight, or capacity.	**Inquiry:** • How can you tell when one thing is bigger than another?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement is used to communicate solutions to real world problems.
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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

1. Recognition and creation of shapes and awareness of position in space.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Describe and name common shapes found in the natural environment. b. Arrange, order, and position objects.	**Inquiry:** • How do we describe where something is? • How do you see common shapes around you?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model geometric relationships. • Students use different representations of geometric relationships to communicate solutions to real world problems.
	Nature of Mathematics: • Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. • Students collaborate to monitor, assess, and make sense of their own thinking and that of others. • Students make and test conjectures, generalize results, and make connections among ideas, strategies, and solutions involving geometric characteristics and relationships.

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Content Area: Mathematics
Standard: Shape, Dimension, and Geometric Relationships

Prepared Graduate Competencies:

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PreK Expectations

Concepts and Skills students know include:

2. Measurement of time and quantity.

Evidence Outcomes **Students can:**	**21st Century Skills and Readiness Competencies**
a. Describe the order of common events. b. Group objects according to their size, length, height, weight, and and size using standard and non-standard forms of measurement.	**Inquiry:** • How do we know how big something is? • How can the order of events be described?
	Applying Mathematics in Society and Using Technology: • Students recognize problems, devise strategies, justify their methods, and revise and refine their strategies in situations involving geometric characteristics and relationships. • Students strategically use technology to explore, organize and communicate mathematical concepts involving geometric relationships. • Students use real world contexts to model measurement. • Students use measurement is used to communicate solutions to real world problems.
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Colorado Department of Education
Office of Standards and Assessments
201 East Colfax Ave. • Denver, CO 80203 • 303-866-6929
www.cde.state.co.us